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1 NUMBER SYSTEM

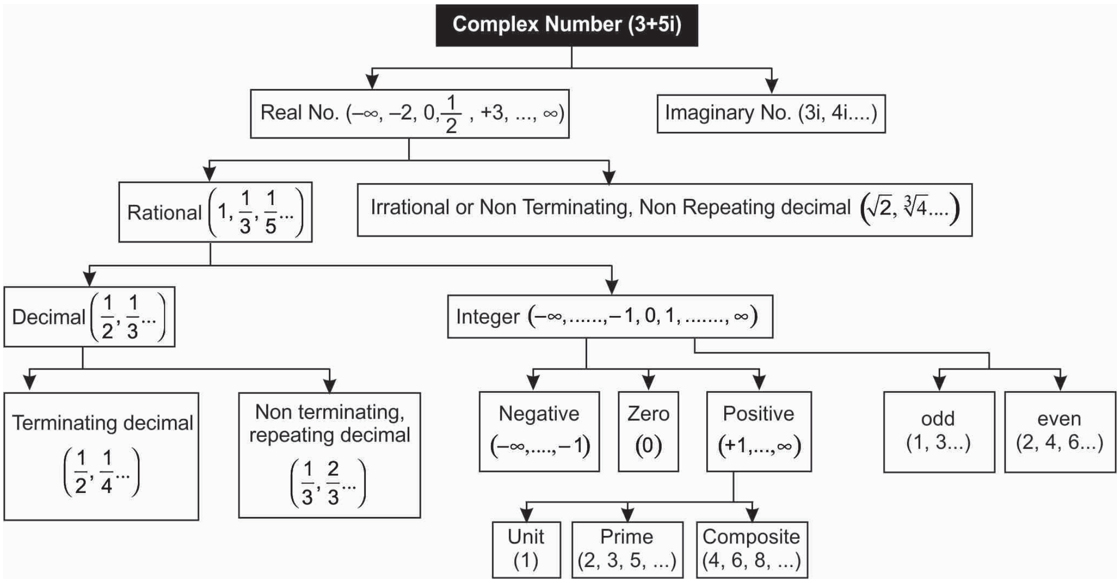
Numbers : A number is denoted by a group of digits, called numeral.
For denoting a numeral 843215696 can be represented as

Ten Crores 10^8	Crores 10^7	Ten Lacs (Millions) 10^6	Lacs 10^5	Ten Thousands 10^4	Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0
8	4	3	2	1	5	6	9	6

TYPES OF NUMBERS

1. **Natural Numbers :** Counting numbers are called natural numbers.
$$N = \{1, 2, 3, 4, 5, ...\}$$
2. **Whole Numbers :** All counting numbers and 0 form the set of whole numbers.
$$W = \{0, 1, 2, 3, 4, 5, ...\}$$
3. **Integers :** All counting numbers, zero and negative of counting numbers form the set of Integers.
$$I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$$
4. **Even Numbers :** The number which is divisible by 2 is called even number. *e.g.,* 2, 4, 12, 28 etc.
5. **Odd Numbers :** The number which is not divisible by 2 is called odd number. *e.g.,* 1, 3, 5, 7 etc.
6. **Prime Numbers :** A number is called a prime number if it has exactly two factors, namely itself and 1. *e.g.,* 2, 5, 11, 19, 23 etc.
7. **Composite number :** The natural number which are not prime, are called composite numbers. *e.g.,* 4, 9, 15, 18, 27 etc.
8. **Rational Numbers :** A rational number is a number that can be put in the form $\frac{p}{q}$ where p and q are both integers and $q \neq 0$ *e.g.,* $7, \frac{-9}{5}, \frac{-2}{7}, \frac{1}{4}, 0$ etc.
9. **Irrational Numbers :** An irrational number is a number that cannot be put in the form $\frac{p}{q}$ where p and q are both Integers and $q \neq 0$ *e.g.,* $\sqrt{7}, \sqrt{11}, \sqrt{13}$ etc.
10. **Real Numbers :** All those numbers which are either rational or irrational. *e.g.,* $\frac{12}{17}, \frac{19}{21}, \sqrt{5}, 5+\sqrt{3}$ etc.
11. **Twin Primes :** Two prime numbers which differ by 2 are called twin primes. *e.g.,* 3, 5 ; 5, 7 ; 11, 13 ; are some pairs of twin primes.

NUMBER TREE



TESTS FOR DIVISIBILITY OF NUMBERS

(i) Divisibility by 2 : A number is divisible by 2 if its units digit is 0, 2, 4, 6 or 8 *e.g.,* 130, 244, 566, 278 etc.

(ii) Divisibility by 3 : A number is divisible by 3 if the sum of its digits is multiple of 3.

e.g., (a) 123 : $1 + 2 + 3 = 6$ which is the multiple of three hence the number is divisible by 3.

(b) 89612 : $8 + 9 + 6 + 1 + 2 = 26 = 2 + 6 = 8$ is not a multiple of three hence the number is not divisible by 3.

(iii) Divisibility by 4 : If the number formed by last two digits is divisible by 4. *e.g.,* 1132, 1312, 1400, 1348 etc.

(iv) Divisibility by 5 : If number unit's digits is either 0 or 5. *e.g.,* 100, 205, 315 etc.

(v) Divisibility by 6 : If number is divisible by both 2 and 3. *e.g.,* 54, 96 etc.

(vi) Divisiblility by 7 : For 7 we need to have – osculator – 2.

e.g., 112 divisible by 7 ?

Step 1. $\underline{11}\ \underline{2} = 11 - 2 \times 2 = 7$ as 7 is divisible by 7 the number is also divisible by 7.

Try for 2961 divisible by 7 ?

(vi) Divisibility by 8 : If number formed by its last three digits is divisible by 8. *e.g.,* 1864, 1024, 2008 and 5000 etc.

(vii) Divisibility by 9 : If the sum of numbers digits is a multiple of 9. *e.g.,* 23409, 454554, 66636 etc.

Example 1. Find the least value of * for which 7 * 5462 is divisible by 9.

Sol. Let the required value be p then

$(7 + p + 5 + 4 + 6 + 2) = (24 + p)$ is divisible by 9.

$\therefore\ p = 3$

Example 2. If the number 653 *xy* is divisible by 90, then find the value of $(x + y)$?

Sol. $90 = 10 \times 9$

Clearly 653 xy is divisible by 10, so $y = 0$

Now, 653 x 0 is divisible by 9

So, $(6 + 5 + 3 + x + 0) = (14 + x)$ is divisible by 9

So, $x = 4$

$\therefore\ x + y = 4 + 0 = 4$

(viii) Divisibility by 10 : If number's unit's digit is zero. *e.g.,* 50, 80, 100, 1310 etc.

(ix) Divisibility by 11 : If the difference of the sum of no's digits in even places and the sum of its digits in odd places is either 0 or a multiple of 11. *e.g.,* 909183, 540045, 184712 etc.

FACTS ABOUT ODD AND EVEN NUMBERS

odd $\pm$ odd = even	odd $\times$ odd = odd
odd $\pm$ even = odd	odd $\times$ even = even
even $\pm$ even = even	even $\times$ even = even

FORMULA FOR DIVISION OF WHOLE NUMBERS

Dividend = Divisor × Quotient + Remainder

NUMBER OF ZEROS OF A MATHEMATICAL EXPRESSION

Zeros are formed by the combination of 2×5, the total number of pairs of 2 and 5 makes zeros in an expression. (Apart from multiples of 10)

To find the number of 2's and 5's we need to factorize the given mathematical expression.

It is to be noted here that in factorial forms the number of 5's will always be lesser than 2's. Hence, we just need to count the number of 5's.

Example 3. Find the number of zeros in 6!

Sol. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (3 \times 2) \times 5 \times 2 \times 2 \times 3 \times 2 \times 1$

This expression contains only one pair of 5 and 2, so only one zero.

REMAINDER THEOREM

Example 4. Find the remainder when 17 × 23 is divided by 12

Sol. $(12+5)\times(12+11) / 12$

In this type of questions remainder only depends on the last term

=> $5\times11 / 12 \Rightarrow 7$

NEGATIVE REMAINDER

Consider the following case

$14 \times 15 /8 \Rightarrow 6\times7 /8 \Rightarrow 2$

However here we can use negative remainder

When 14 is normally divided by 8 remainders is seen as +6, however, there might be a time where negative remainder can be more useful.

Remainders, by concepts, are always positive, hence when we divide -27 by 5 we say that remainder is 3 and not (–2). however we can use the negative remainder to ease our calculations.

Thus above example may be solved as $-2 \times -1 \Rightarrow 2$, which is same but less calculative.

If in this case answer comes out to be negative you need to subtract it with original number. e.g. $(62 \times 63 \times 64)/66$

$\Rightarrow (-4 \times -3 \times -2) \Rightarrow -24$

Hence, required remainder will be $66 - 24 = 42$

HOW TO FIND REMAINDER WHILE DEALING WITH LARGE POWERS:

1) If the expression can be expressed in the form of $\{(ax+1)^n\}/a$, the remainder will become 1 directly, the value of the power does not matter here.

e.g. $46^{32624}/9 \Rightarrow 1^{32624} \Rightarrow 1$

2) If the expression can be expressed in the form of $\{(ax-1)^n\}/a$, then if n is even then, the remainder will be +1, or n is odd then, the remainder will be -1.

e.g. $35^{243}/9 \Rightarrow (-1)^{243} \Rightarrow -1$

$\Rightarrow$ hence remainder $= -1 + 9 = 8$

Example 5. Find the remainder when 43^{197} is divided by 7

Sol. $43^{197}/7 \Rightarrow 1^{197} \Rightarrow 1$

Example 6. A number when divided by 899 gives a remainder of 63. If the same number is divided by 29, then what will be the remainder ?

Sol.

$$\begin{aligned} \text{Number} &= \text{Divisor} \times \text{Quotient} + \text{Remainder} \\ &= 899 \times x + 63 \\ &= 31 \times 29 \times x + 29 \times 2 + 5 \\ &= 29\,(31x + 2) + 5 \end{aligned}$$

$\therefore$ The remainder when the number is divided by 29 is 5

SMART WAY

Number = Divisor × Quotient + Remainder
= 899 × 1 + 63 = 962
(Always take Quotient as 1)
No. Divide by 29 we gt remainder as 5)

Example 7. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, determine the dividend.

Sol. Let the quotient be Q and the remainder be R

$\therefore$ Divisor $= 5 \times 46 = 230$

$$\text{Quotient} = \frac{230}{10} = 23$$

$\therefore$ Dividend = Divisor × Quotient + Remainder

$= 230 \times 23 + 46 = 5290 + 46 = 5336$

Example 8. Find the largest number which divides 25, 73 and 97 leaving an equal remainder in each case.

Sol. $\because$ Number = Divisor × Quotient + Remainder

$25 = 24 \times 1 + 1$ ...(i)

$73 = 24 \times 3 + 1$ ...(ii)

$97 = 24 \times 4 + 1$ ...(iii)

$\therefore$ 24 is the largest number which divides the given three numbers leaving 1 as remainder in each case.

Example 9. On dividing a number by 68, we get 269 as quotient and 0 as remainder on dividing the same number by 67, what will be the remainder ?

Sol. Number = 269 × 68 + 0 = 18292

```
67) 18292 (273          269
    134                 ×68
    ----               -----
     489               2152
     469              1614×
     ----             -----
      202             18292
      201
      ---
        1
```

SMART WAY

As per the question the number is a multiple of 68 so on dividing 68m by 67 we always gets remainder as 1.

$\therefore$ Required Remainder = 1

Example 10. What least number must be subtracted from 1672 to obtain a number which is completely divisible by 17 ?

Sol.

```
17) 1672 (98
    153
    ----
     142
     136        Number to be subtracted = 6
     ---
       6
```

Example 11. What least number must be subtracted from 13601, so that number is divisible by 87 ?

Sol.

```
87) 13601 (156
    87
    ----
    490
    435
    ----
     551
     522
     ----
      29
```

$\therefore$ Required number = 29

Example 12. What is the smallest 5-digit number exactly divisible by 41 ?

Sol. The smallest 5-digit number = 10000

$\therefore$ Required number = 10000 + (41 – 37) = 10004

```
41) 10000 (243
     62
    ----
    180
    164
    ----
     160
     123
    ----
      37
```

Natural No. Series	Sum	Average
$1 + 2 + ... + n$	$\frac{n(n+1)}{2}$	$\frac{n+1}{2}$
$1^2 + 2^2 + ... + n^2$	$\frac{n(n+1)(2n+1)}{6}$	$\frac{(n+1)(2n+1)}{6}$
$1^3 + 2^3 + ... + n^3$	$\left[\frac{n(n+1)}{2}\right]^2$	$n\left[\frac{n+1}{2}\right]^2$

Odd No. Series	Sum	Average
$1 + 3 + ... + x$	n^2	n
$1^2 + 3^2 + ... + x^2$	$\frac{n(4n^2-1)}{3}$	$\frac{4n^2-1}{3}$
$1^3 + 3^3 + ... + x^3$	$n^2(2n^2 - 1)$	$n(2n^2 - 1)$
	where, $n = \frac{x+1}{2}$	

Even No. Series	Sum	Average
$2 + 4 + ... + x$	$n(n + 1)$	$n + 1$
$2^2 + 4^2 + ... + x^2$	$\frac{2}{3}n(n+1)(2n+1)$	$\frac{2}{3}(n+1)(2n+1)$
$2^3 + 4^3 + ... + x^3$	$2[n(n + 1)]^2$	$2n(n + 1)^2$
	where, $n = \frac{x}{2}$	

CONCEPT OF A.P. & G.P.

Arithmetic Series :–

An Arithmetic series is one in which successive numbers are obtained by adding (or subtracting) a fixed number to previous no. for *e.g.,* 3, 5, 7, 9, 11**----------**

d (common difference) = 5 – 3 = 2 = 7 – 5 = 2

$a \rightarrow$ first term $n \rightarrow$ number of terms

$l \rightarrow$ last term $= a + (n - 1)d$

sum of n terms $= \frac{n}{2}[a + l]$

Geometric Series :-

Geometric Series is one in which each successive number is obtained by multiplying (or dividing) a fixed number by the previous number.

For example 4, 8, 16, 32, 64-----------

r (common ratio) $= \frac{16}{8} = \frac{8}{4} = 2$

$a \rightarrow$ first term

Sum of n terms $= \frac{a(r^n - 1)}{r - 1}$

Example 13. How many natural numbers between 17 and 80 are divisible by 6 ?

Sol. These numbers are 18, 24, 30, 36,...., 78 which is an A.P.

Here, $a = 18$, $d = 24 - 18 = 6$ and $l = 78$

$l = a + (n - 1)\, d$

$78 = 18 + (n - 1)\, 6$

$\Rightarrow (n - 1) \times 6 = 60 \Rightarrow n - 1 = 10 \Rightarrow n = 11$

$\therefore$ Required numbers = 11

SMART WAY

Series is 18, 24, 30, 36------78

Your Series can be written as 6 (3, 4, 5, 6----13)

Total numbers are = 13 – 3 + 1 = 11

Alsways remember to add 1.

Example 14. Find the sum of all even natural numbers less than 75.

Sol. Sum = 2 + 4 + 6 + ... + 74 which is an A.P.

Here, $a = 2$, $d = 4 - 2 = 2$, $l = 74$, $n = 37$

$\therefore$ Sum $= \frac{n}{2}(a + l) = \frac{37}{2} \times (2 + 74) = 37 \times 38 = 1406$

Example 15. What is the sum of the numbers between 1 to 500 ?

Sol. Sum of numbers from 1 to 500 $= \frac{500\,(500+1)}{2}$ [$\because$ Sum of numbers from 1 to $n = \frac{n\,(n+1)}{2}$]

$$= \frac{500 \times 501}{2} = \frac{250500}{2} = 125250$$

Example 16. Find the sum $(2 + 2^2 + 2^3 + 2^4 + ... + 2^{10})$

Sol. This is an G.P.

$$S = \frac{a\,(r^n - 1)}{r - 1} = \frac{2\,(2^{10} - 1)}{2 - 1} = 2 \times 1023 = 2046$$

NUMBER OF FACTORS OF A COMPOSITE NUMBER

In general, for any composite number C, which can be expressed as C $= a^m \times b^n \times c^p$ ------------ where a, b, c---------are all prime factors and m, n, p ---------------- are positive integers, the number of factors is equal to $(m + 1)\,(n + 1)\,(p + 1)$------------------- .

Number of prime factors $= m + n + p$ --------------