## PIPES AND CISTERNS

## INTRODUCTION

Concept of Pipes and cisterns are almost the same as Time and work problems. If a pipe fills a tank in 5 hrs, then the pipe fills $\frac{1}{5}$ th of the tank in 1 hr . There are outlets as well as inlets in this topic; it is the difference in Pipes and Cisterns problems. These outlets perform negative work too. The rest of the process is almost same in these two topics.

Inlet: A pipe which fills a tank or a cistern or a reservoir.
A pipe takes $F$ hours to fill up the tank. Then $\frac{1}{F}$ parts of the tank will be filled in 1 hour.
Outlet: A pipe which empties a tank or cistern or reservoir.
A pipe takes $x$ hours to empty the tank. Then part emptied in 1 hour $=\frac{1}{x}$
Ex. Two pipes, pipe $X$ and pipe $Y$ can fill a cistern in 40 hours and 60 hours respectively while working alone. If both pipes are opened together, then the time taken to fill the cistern is?
Sol: Pipe $X=\frac{1}{40}$
Pipe $Y=\frac{1}{60}$
$=\frac{1}{40}+\frac{1}{60}$
$=\frac{5}{120}=\frac{1}{24}$
$=24$ hours
Ex: Two pipes, Pipe $X$ and Pipe $Y$ can empty a cistern in 24 hours and 40 hours respectively while working alone. If both pipes are opened together, then the time taken to empty the cistern is given by
Sol: Pipe $X=-\frac{1}{24}$
Pipe $Y=-\frac{1}{40}$
$=-\frac{1}{24}-\frac{1}{40}$
$=-\frac{8}{120}=-\frac{1}{15}$
$=15$ Hours
Note: "-"sign denotes negative work.
Ex: Two pipes, pipe $X$ can fill a tank in 20 min and pipe $Y$ can empty a tank in 30 hours while working alone. If both pipes are opened together, then the time taken to fill/empty the tank is?
Sol: Pipe $X=\frac{1}{20}$
Pipe $Y=-\frac{1}{30}$
$=\frac{1}{20}-\frac{1}{30}$
$=\frac{3-2}{60}=\frac{1}{60}$
$=60$ Hours

Ratio concept:
If Pipe $A$ is $\mathbf{3}$ times fast as another pipe $B$, then
Ratio of filling/emptying by Pipe $A$ and $B=3: 1$
Ratio of times taken by A and B to fill/empty $=1: 3$
Ex: If pipe $X$ is 2 times fast as pipe $Y$ and can fill a tank 15 min less than pipe $Y$. Then find the time in which both the pipes can fill the tank together?
Sol: let Pipe $\mathrm{X}=\mathrm{x}$
i.e. $\frac{1}{x}$

Pipe $Y=\frac{1}{2 x}$
$\frac{1}{x}-\frac{1}{2 x}=\frac{1}{15}$
$\frac{1}{x}=\frac{1}{15}$
$X=15 \mathrm{~min}$
$2 \mathrm{x}=30 \mathrm{~min}$
Together $=\frac{1}{15}+\frac{1}{30}=\frac{3}{30}$
$=\frac{1}{10}$
$=10 \mathrm{~min}$

## Alternate concept

If more than one pipe is there and opens one after another at a time turn wise, it is called alternate concept of pipe and cistern problems. In this case, more than one pipe is there to fillempty a tank or a cistern but only one pipe is operating at a time.

Ex. There are 3 pipes A, B and C can fill a tank in 6, 12 and 8 hours respectively. Find the time taken to fill the tank if work is done on alternate basis?
Sol:
A $=6$
B $=12$
C = 8
Total units $=24$
A's units $=4$
B's units $=2$
C's units $=3$
Total $=9$ units
9 units in 3 Hours
18 units in 6 Hours
4 units by A on $7^{\text {th }}$ hour
Remaining $=24-22=2$
Remaining time $=\frac{2}{2}=1$ hour
Total time $=7+1=8$ hours
Ex. Three pipes A, B and C can fill a cistern in 12, 15 and 20 hours respectively. Find the time taken by all the pipes to fill the cistern?
Sol: $\mathrm{A}=\frac{1}{12}$
$B=\frac{1}{15}$
$\mathrm{C}=\frac{1}{20}$
Together $=\frac{1}{12}+\frac{1}{15}+\frac{1}{20}$
$=\frac{12}{60}=\frac{1}{5}$
$=5$ Hours

## Important Questions for Practice

1. There are two Pipes, $A$ and $B$ can fill a container in 8 and 12 hours respectively. And another Pipe $C$ can empty in 6 hours. If all the three pipes are opened together, then the container will be filled in.
(A) 12 hours
(B) 15 hours
(C) 20 hours
(D) 24 hours
(E) None of these
2. Two pipes $A$ \& $B$ can fill a cistern in 18 min and 27 min respectively. If both pipers are opened and $A$ is turned off after $x$ minutes then the cistern will be filled in just 16 min . Find the value of $x$.
(A) 4 min
(B) 5 min
(C) 6 min
(D) 7 min
(E) 8 min
3. A cistern has three pipes $P, Q, R$; Pipe $P$ and $Q$ can fill it in 5 and 6 hr respectively; and pipe $R$ can empty it in 2 hr . If these pipes are opened in order at $7 \mathrm{a} . \mathrm{m}$, at 8 a.m. and 9 a.m., then at what time will the cistern be emptied?
(A) $12: 45 \mathrm{am}$
(B) $1: 15$ p.m
(C) $1: 45 \mathrm{p} . \mathrm{m}$
(D) $2: 15$ p.m
(E) None of these
4. An inlet pipe $A$ can fill water at the rate of 5 litres per minute and a leakage find in the bottom of a tank which can empty the full tank in 8 hours. When the tank is full, the inlet pipe $A$ is opened and due to the leak the tank is empty in 12 hours. What is the capacity of the tank? (In litres)
(A) 6400 litres
(B) 6800 litres
(C) 7000 litres
(D) 7200 litres
(E) None of these
5. Two pipes $P$ and $Q$ can fill a tank in $7 \frac{1}{2}$ hours together. Separately, pipe $Q$ alone can take 8 hours more than pipe $P$. Find the time taken by pipe $Q$ to fill the tank alone.
(A) 12 hours
(B) 15 hours
(C) 20 hours
(D) 24 hours
(E) None of these
6. There are three pipes $A, B$ and $C$ can fill an empty tank in 6 hours. The pipe $B$ is twice as fast as pipe $A$ and pipe $C$ is thrice as efficient as pipe $B$. Find the time taken by Pipe $A$ and Pipe $B$ to fill the tank?
(A) 12 hours
(B) 15 hours
(C) 18 hours
(D) 20 hours
(E) 24 hours
7. A Pipe A can fill a tank in 12 hours and another pipe $B$ can fill it in 16 hours. Both the pipes are opened together at 10 a.m. and pipe $B$ is turned off after some time. The tank is filled at $7 \mathrm{p} . \mathrm{m}$., and then finds the time at which the pipe $B$ turned off?
(A) 12 p.m.
(B) 1:00 p.m.
(C) 2:00 p.m.
(D) 2:30 p.m.
(E) 3:00 p.m.
8. A pump can fill a tank in 4 hours. A leakage is found in the bottom of the tank which empty 2 litres of water per minute. Because of the leakage it was taking half an hour more than the usual time.
What is the capacity of the tank?
(A) 3432 litres
(B) 3884 litres
(C) 4230 litres
(D) 4320 litres
(E) 4560 litres
9. There are two filling pipes $A$ and $B$ can fill the tank in 6 hours and 8 hours respectively. And there is also a waste pipe C which can empty the tank at the rate of 5 litres per minute. All the three pipes are opened together, the tank be filled in 12 hours. Find the capacity of the tank.
(A) 1220 litres
(B) 1440 litres
(C) 1680 litres
(D) 1850 litres
(E) 2080 litres
10. There are three pipes $A, B$ and $C$ attached to a tank. Pipe $A$ and pipe $B$ can fill it in 6 hours and 8 hours respectively and pipe $C$ can empty it in 3 hours. If pipes are opened in order at 9 a.m., 10 a.m. and 11 p.m. respectively, when will the tank be empty?
(A) 3:00 p.m.
(B) 5:30 p.m.
(C) 7:00 p.m.
(D) $8: 30 \mathrm{p} . \mathrm{m}$.
(E) 10:00 p.m.

## Solutions

## 1. Answer is option D <br> Explanation:

Pipe $A=8$ hours (fill)
Pipe $B=12$ hours (fill)
Pipe $C=6$ hours (empty)
Lcm $=24$
Units fill/empty
Pipe $A=3$ units (fill)
Pipe $B=2$ units (fill)
Pipe $C=4$ units (empty)
Together =5-4 = 1 unit (fill)
Time $=\frac{24}{1}=24$ hours

## 2. Answer is option C <br> Explanation:

Pipe $\mathrm{A}=18 \mathrm{~min}$
Pipe $B=24 \mathrm{~min}$
LCM $=72$ units
Pipe $A=4$ units (fill)
Pipe $B=3$ units (fill)
Together = 7 units (fill)
B worked for whole the time
$4 \times x+16 \times 3=72$
$4 x=72-48$
$x=\frac{24}{4}$
$x=6$ min

## 3. Answer is option B

Explanation:
Pipe $P=5$ hours (fill)
Pipe $Q=6$ hours (fill)
Pipe $R=2$ hours (empty)
LCM = 30 units
Units per hour
Pipe $P=6$ (fill)
Pipe $Q=5$ (fill)
Pipe $R=15$ hours (empty)
Together $=15-11=4$ units
Pipe $P$ fill $=6 \times 2=12$ units
Pipe $Q$ fill $=5 \times 1=5$ units
Total units fill till 9 a.m. $=12+5=17$
Together they empty 5 units per hour.
Time to empty $=\frac{17}{4}$
$=4$ hours 15 min
Empty at 1:15p.m.

Pipe $A=4$
Pipe B = 3
Tank filled in 9 hours.
As per question,
Pipe A worked for all the time.
Let Pipe B work $=x$
$4 \times 9+3 \times x=48$
$36+3 x=48$
$3 x=12$
$x=4$ hours
Time $=2$ p.m.

## 8. Answer is option D

Explanation:
Filling pump $=4$ hours
Let leakage $=x$
With leakage $=\frac{9}{2}$ hours
$\frac{1}{x}=\frac{1}{4}-\frac{2}{9}$
$\frac{1}{x}=\frac{9-8}{36}$
$x=36$ hours
Leak 2 litres per min.
Time $=36$ hours
Capacity $=36 \times 60 \times 2=4320$ litres

## 9. Answer is option B

Explanation:
Pipe $A=6$ hours
Pipe $B=8$ hours
Pipe $A+B-C=12$ hours
LCM = 24 units
Efficiency
Pipe $A=4$ units
Pipe $B=3$ units
Pipe $A+B+C=2$ units
Pipe $C=-5$ units
Time taken by pipe $C=\frac{24}{-5}$
$=\frac{24}{-5}$
Capacity $=\frac{24}{5} \times 60 \times 5=1440$ litres

## 10. Answer is option E <br> Explanation:

Pipe $A=6$ hours
Pipe $B=8$ hours
Pipe $C$ (empty) $=3$ hours
LCM $=24$ units

Efficiency
Pipe $A=4$ units
Pipe $B=3$ units
Pipe $C$ (empty) $=-8$ units
At 9 a.m. Pipe A opened
$4 \times 1=4$ units
At 10 a.m. Pipe B opened
$7 \times 1=7$ units
Total work done till 11 a.m. $=7+4=11$ units
After 11 a.m. all worked together.
$7-8=-1$ unit
Time taken by all to empty $=\frac{11}{1}$
= 11 hours
Time $=10: 00$ p.m.
make Yexam

