## RATIO AND PROPORTION

## What is Ratio?

Ratio is a term used to compare two similar quantities in the same units. The ratio of two terms ' A ' and ' B ' is denoted by $\mathrm{a}: \mathrm{b}$. In ratio $\mathrm{a}: \mathrm{b}$, we can say that a as the first term or antecedent and $b$, the second term or consequent.
In general, the ratio of a number $a$ to $a$ number $b$ is defined as the quotient of the numbers $a$ and $b i . e$. a/b.

Example: The ratio of 20 km to 80 km is 20:80, which is $1: 4$ or $\frac{1}{4}$, where 1 is called the antecedent and 4 the consequent.

Note: Ratios and Fractions are same; the only difference is that ratio is a unit less quantity while fraction is not.

## Compound Ratio

Ratios are compounded by multiplying together the fractions, which denote them; or by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent. The compound of $\mathrm{a}: \mathrm{b}$ and $\mathrm{c}: \mathrm{d}$
is $\frac{a \times c}{b \times d}$ i.e. ac : bd.
Example 1: If the ratio between the first number and second number be $1: 3$ and that between the second and third be $5: 4$, then find the combined ratio.
Sol: The ratio among the three numbers is
Let $\mathrm{A}: \mathrm{B}=1: 3$
Let $\mathrm{B}: \mathrm{C}=5: 4$
A:B:C
1:3
5:4
$5: 15: 12$

Example 2: Divide 774 into two parts such that the ratio is $2: 7$.
Sol: First part $=2 \times \frac{774}{2+7}=2 \times \frac{774}{9}=172$
Second part $=7 \times \frac{774}{2+7}=7 \times \frac{774}{9}=602$

## Consider the two ratios:

1st ratio 2nd ratio
8: 16 12: 24
Since 8 is one-half of 16 , and 12 is one-half of 24 , the two ratios are equal. The equality of ratio is called proportion.
The number 8, 16, 12 and 24 are said to be in proportion.
The proportion may be written as
$8: 16:: 12: 24(8$ is to 16 as 12 is to 24 )
Or, $8: 16=12: 24$
Or, $8 / 16=12 / 24$

If four quantities be in proportion, the product of the extremes is equal to the product of the means.
Example: Let the four quantities $3,4,9$ and 12 be in proportion,
We have
$\frac{3}{4}=\frac{9}{12}$
Cross multiply numbers
$3 \times 12=4 \times 9$
Three quantities of the same kind are said to be in continued proportion when the ratio of the first to the second is equal to the ratio of the second to the third.

The second quantity is called mean proportion between the first and third; and the third quantity is called the third proportional to the first and second.

Example: Find the fourth proportional to the numbers 6, 8 and 15
Sol: If $x$ be the fourth proportional, then $6: 8=15: x$

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\therefore \mathrm{x}=\frac{(8 \times 15)}{6}=20
$$

## Important Questions for Practice

1. Amit, Bitu and Chandu started a business with investment in the ratio $6: 5: 7$ respectively. After one year Chandu withdrew $50 \%$ of his capital and Bitu increased his capital by $40 \%$ of his investment. After two years, what should be the ratio of distribution of the profit among Amit, Bitu and Chandu respectively?
(A) $3: 3: 4$
(B) $6: 5: 4$
(C) $7: 5: 6$
(D) $8: 8: 7$
(E) None of these
2. Three men A, B and C start a business together. They invest Rs. 32000, Rs. 24000 and Rs. 40000 respectively in the beginning. After 4 months. A took out Rs. 4000 and $C$ took out Rs. 5000. They get a profit of Rs. 10800 at the end of the year. B's share in the profit is
(A) Rs. 2550
(B) Rs. 2760
(C) Rs. 2880
(D) Rs. 3220
(E) None of these
3. Rohit invested certain amounts in two different schemes ' $A$ ' \& ' $B$ ' Scheme ' $A$ ' offers simple interest at $12 \%$ p.a. and Scheme ' $B$ ' of offers compound interest at $10 \%$ p.a. Interest accrued on the amount invested in Scheme A in 2 years was Rs. 3,600 and the total amount invested was Rs 35,000. What was the interest accrued on the amount invested in Scheme B?
(A) Rs. 3,600
(B) Rs. 4,200
(C) Rs. 4,800
(D) Cannot be determined
(E) None of these
4. $X, Y$ and $Z$ were sharing profits in the ratio $4: 3: 2$. $Y$ retired from the firm and $X$ and $Z$ decide to share profits in the ratio $3: 2$. Calculate the gaining ratio.
(A) $7: 8$
(B) $5: 9$
(C) $4: 7$
(D) $5: 8$
(E) None of these
5. A, B and C are three partners. They altogether invested Rs 14000 in business. At the end of the year, A got Rs 337.50 , $B$ Rs 1125 and C Rs 637.50 as profit. The difference between the investments of $A$ and $B$ was
(A) Rs 2200
(B) Rs 3200
(C) Rs 4200
(D) Rs 5250
(E) None of these
6. The investment of Ravi is twice as that of Amit and thrice as that of Sumit. Amit invested for thrice the months than Ravi and twice the months than Sumit. Who will earn the highest profit?
(A) Ravi
(B) Amit
(C) Sumit
(D) Cannot be determined
(E) None of these
7. A, B and C came into a partnership and agree to share profit in the ratio $4: 6: 9$ respectively. A and C received a total of Rs 3,900 . What is the amount that $C$ and $B$ got together?
(A) Rs 3,000
(B) Rs 3,600
(C) Rs 4,500
(D) Rs 5,000
(E) None of these
8. Rohit and Rahul entered into a partnership investing Rs. 10000 and Rs. 8000 rep. After 3 months Rohan joined them with an investment of Rs.12000. What is the share of Rahul in a half yearly profit of Rs.4800?
(A) Rs. 1600
(B) Rs. 2000
(C) Rs. 2400
(D) Rs. 2500
(E) None of these
9. Shivaye and Om enter into a partnership with Rs 50000 and Rs 60000 respectively. Jai joins them after x months contribuing Rs 70000 and Om leaves x months before the end of the year. If they share the profit in the ratio of $20: 18: 21$, then the value of $x$ is
(A) 3
(B) 4
(C) 6
(D) 9
(E) None of these
10. A man left one-half of the capital for his wife, one-third to his son and the remainder to his daughter. His daughter and wife have the share worth Rs $3,00,000$. Find the share of the son.
(A) Rs 90,000
(B) Rs 1,25,000
(C) Rs 1,50,000
(D) Rs 3,00,000
(E) None of these

## Solutions

1. Answer is option D

Explanation:
Total capital of Amit
$=6 \mathrm{x} \times 24=144 \mathrm{x}$
Total equivalent capital of Bitu for 1 month
$=5 \mathrm{x} \times 12+7 \mathrm{x} \times 12=144$
Total equivalent capital of Chandu
$=7 \mathrm{x} \times 12+3.5 \mathrm{x} \times 12=$ Rs. 126 x
Required ratio $=$ Amit : Bitu : Chandu
$=144 x: 144 x: 126 x$
$=8: 8: 7$

## 2. Answer is option C

Explanation:
Ratio of equivalent capitals
$=(32000 \times 4+28000 \times 8): 24000 \times 12:(40000 \times$
$4+35000 \times 8$ )
$=352000: 288000: 440000$
= $352: 288: 440$
= $44: 36: 55$
Sum of ratios $=44+36+55=135$
B's share $=36$
$\frac{10800}{135} \times 36=$ Rs. 2880

## 3. Answer is option B

## Explanation:

Simple interest is always same every year.
$12 \%+12 \%=24 \%$ of Scheme A
$24 \%$ of Scheme A = 3600
$100 \%$ of Scheme A $=\frac{3600}{24} \times 100$
$=15000$
Investment in Scheme B
$=35000-15000=20000$
Interest on Scheme B
$10 \%+10 \%+\frac{10 \times 10}{100}=21 \%$
$21 \%$ on 20000
Interest $=20000 \times \frac{21}{100}$
$=4200$

## 4. Answer is option A <br> Explanation:

X: Y: Z
4:3:2
After Y's retirement
X: Y

3:2
Gaining Ratio
$\begin{array}{l:l}\mathrm{X} & \mathrm{Z} \\ \left(\frac{3}{5}-\frac{4}{9}\right) & :\left(\frac{2}{5}-\frac{2}{9}\right)\end{array}$
7:8

## 5. Answer is option D

Explanation:
Ratio of investments of A, B and C
= 337.50 : 1125 : 637.50
= $27: 90: 51$
= 9 : $30: 17$
Let the investments of $A, B$ and $C$ be $9 x, 30 x$ and 17 x respectively
$9 \mathrm{x}+30 \mathrm{x}+17 \mathrm{x}=14000$
$x=250$
So, required difference
$=30 \mathrm{x}-9 \mathrm{x}=21 \mathrm{x}$
$21 \mathrm{x}=\mathrm{Rs} 5250$
6. Answer is option B

Explanation:

Time 2y 6y $\quad 3 y$

Profit 12xy 18xy 6xy
Final profit sharing ratio
Ravi Amit Sumit
Hence, Amit will earn the highest profit

## 7. Answer is option C

## Explanation:

A: B:C
$4 \mathrm{x}: 6 \mathrm{x}: 9 \mathrm{x}$
A and C received $=3900$
$\mathrm{A}+\mathrm{C}=13 \mathrm{x}$
$13 \mathrm{x}=3900$
$\mathrm{x}=300$
$B+C=15 x$
$=15 \times 300$
$=4500$
8. Answer is option A

Explanation:
Rohit $=10000 \times 6=60000$
Rahul $=8000 \times 6=48000$
Rohan $=12000 \times 3=36000$
Profit sharing ratio
Rohit Rahul Rohan
600004800036000
$5 \quad: 4 \quad: 3$
Rahul's share $=4800 \times \frac{4}{12}=1600$

## 9. Answer is option A

Explanation:
Shivaye there for whole time = 12 months Invested $=50000$

Om's investment $=60000$
Time $=12-\mathrm{x}$ months
Jai's investment $=70000$
Time $=12-\mathrm{x}$ months
Take any two
$\frac{\text { Shivaye }}{O m}=\frac{20}{18}$
$\frac{50000 \times 12}{60000 \times(12-x)}=\frac{20}{18}$
$9=12-\mathrm{x}$
$\mathrm{x}=12-9$
$\mathrm{x}=3$

## 10. Answer is option C Explanation:

Let his total capital $=\mathrm{x}$
His wife's share $=\frac{1}{2} x$
His son's share $=\frac{1}{3} x$
His daughter's share $=\mathrm{x}-\left(\frac{1}{2} x+\frac{1}{3} x\right)$
$=\frac{1}{6} x$
Wife and daughter's share $=\frac{1}{6} x+\frac{1}{2} x$
$=\frac{2}{3} x=300000$
$=\mathrm{x}=450000$
Son's share $=450000 \times \frac{1}{3}$
= 150000

