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Advanced Maths

COURSE BOOK

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NUMBER SYSTEM

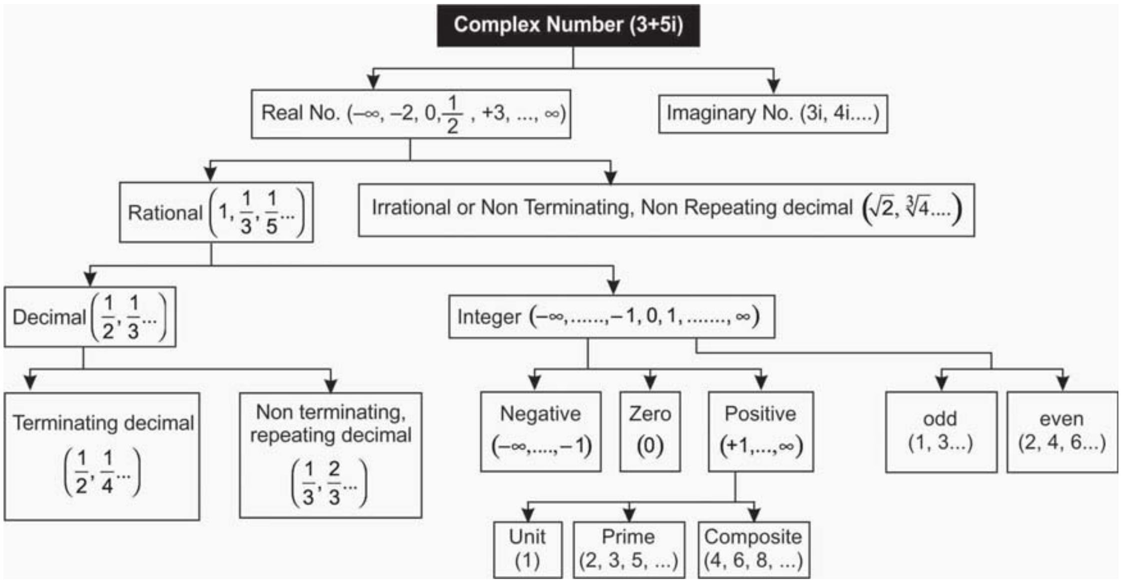
Numbers : A number is denoted by a group of digits, called numeral.
For denoting a numeral 843215696 can be represented as

Ten Crores 10^8	Crores 10^7	Ten Lacs (Millions) 10^6	Lacs 10^5	Ten Thousands 10^4	Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0
8	4	3	2	1	5	6	9	6

TYPES OF NUMBERS

1. **Natural Numbers :** Counting numbers are called natural numbers.
   N = {1, 2, 3, 4, 5,....}
2. **Whole Numbers :** All counting numbers and 0 form the set of whole numbers.
   W = {0, 1, 2, 3, 4, 5,...}
3. **Integers :** All counting numbers, zero and negative of counting numbers form the set of Integers.
   I = {–4, –3, –2, –1, 0, 1, 2, 3, 4,....}
4. **Even Numbers :** The number which is divisible by 2 is called even number. *e.g.,* 2, 4, 12, 28 etc.
5. **Odd Numbers :** The number which is not divisible by 2 is called odd number. *e.g.,* 1, 3, 5, 7 etc.
6. **Prime Numbers :** A number is called a prime number if it has exactly two factors, namely itself and 1. *e.g.,* 2, 5, 11, 19, 23 etc.
7. **Composite number :** The natural number which are not prime, are called composite numbers. *e.g.,* 4, 9, 15, 18, 27 etc.
8. **Rational Numbers :** A rational number is a number that can be put in the form $\frac{p}{q}$ where p and q are both integers and $q \neq 0$ *e.g.,* $7, \frac{-9}{5}, \frac{-2}{7}, \frac{1}{4}, 0$ etc.
9. **Irrational Numbers :** An irrational number is a number that cannot be put in the form $\frac{p}{q}$ where p and q are both Integers and $q \neq 0$ *e.g.,* $\sqrt{7}, \sqrt{11}, \sqrt{13}$ etc.
10. **Real Numbers :** All those numbers which are either rational or irrational. *e.g.,* $\frac{12}{17}, \frac{19}{21}, \sqrt{5}, 5+\sqrt{3}$ etc.
11. **Twin Primes :** Two prime numbers which differ by 2 are called twin primes. *e.g.,* 3, 5 ; 5, 7 ; 11, 13 ; are some pairs of twin primes.

NUMBER TREE



TESTS FOR DIVISIBILITY OF NUMBERS

(i) Divisibility by 2 : A number is divisible by 2 if its units digit is 0, 2, 4, 6 or 8 *e.g.,* 130, 244, 566, 278 etc.

(ii) Divisibility by 3 : A number is divisible by 3 if the sum of its digits is multiple of 3.

e.g., (a) 123 : $1 + 2 + 3 = 6$ which is the multiple of three hence the number is divisible by 3.

(b) 89612 : $8 + 9 + 6 + 1 + 2 = 26 = 2 + 6 = 8$ is not a multiple of three hence the number is not divisible by 3.

(iii) Divisibility by 4 : If the number formed by last two digits is divisible by 4. *e.g.,* 1132, 1312, 1400, 1348 etc.

(iv) Divisibility by 5 : If number unit's digits is either 0 or 5. *e.g.,* 100, 205, 315 etc.

(v) Divisibility by 6 : If number is divisible by both 2 and 3. *e.g.,* 54, 96 etc.

(vi) Divisiblility by 7 : For 7 we need to have – osculator – 2.

e.g., 112 divisible by 7 ?

Step 1. $\underline{11}\ \underline{2} = 11 - 2 \times 2 = 7$ as 7 is divisible by 7 the number is also divisible by 7.

Try for 2961 divisible by 7 ?

(vi) Divisibility by 8 : If number formed by its last three digits is divisible by 8. *e.g.,* 1864, 1024, 2008 and 5000 etc.

(vii) Divisibility by 9 : If the sum of numbers digits is a multiple of 9. *e.g.,* 23409, 454554, 66636 etc.

Example 1. Find the least value of * for which 7 * 5462 is divisible by 9.

Sol. Let the required value be p then

$(7 + p + 5 + 4 + 6 + 2) = (24 + p)$ is divisible by 9.

$\therefore p = 3$

Example 2. If the number 653 xy is divisible by 90, then find the value of $(x + y)$?

Sol. $90 = 10 \times 9$

Clearly 653 xy is divisible by 10, so $y = 0$

Now, 653 x 0 is divisible by 9

So, $(6 + 5 + 3 + x + 0) = (14 + x)$ is divisible by 9

So, $x = 4$

$\therefore x + y = 4 + 0 = 4$

(viii) Divisibility by 10 : If number's unit's digit is zero. *e.g.,* 50, 80, 100, 1310 etc.

(ix) Divisibility by 11 : If the difference of the sum of no's digits in even places and the sum of its digits in odd places is either 0 or a multiple of 11. *e.g.,* 909183, 540045, 184712 etc.

FACTS ABOUT ODD AND EVEN NUMBERS

odd ± odd = even	odd × odd = odd
odd ± even = odd	odd × even = even
even ± even = even	even × even = even

FORMULA FOR DIVISION OF WHOLE NUMBERS

Dividend = Divisor × Quotient + Remainder

Example 3. A number when divided by 899 gives a remainder of 63. If the same number is divided by 29, then what will be the remainder ?

Sol. Number = Divisor × Quotient + Remainder

$= 899 \times x + 63$

$= 31 \times 29 \times x + 29 \times 2 + 5$

$= 29 (31x + 2) + 5$

∴ The remainder when the number is divided by 29 is 5

SMART WAY

Number = Divisor × Quotient + Remainder
= 899 × 1 + 63 = 962
(Always take Quotient as 1)
No. Divide by 29 we gt remainder as 5)

Example 4. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, determine the dividend.

Sol. Let the quotient be Q and the remainder be R

∴ Divisor = 5 × 46 = 230

$$\text{Quotient} = \frac{230}{10} = 23$$

∴ Dividend = Divisor × Quotient + Remainder

= 230 × 23 + 46 = 5290 + 46 = 5336

Example 5. Find the largest number which divides 25, 73 and 97 leaving an equal remainder in each case.

Sol. ∵ Number = Divisor × Quotient + Remainder

25 = 24 × 1 + 1 ...(i)

73 = 24 × 3 + 1 ...(ii)

97 = 24 × 4 + 1 ...(iii)

∴ 24 is the largest number which divides the given three numbers leaving 1 as remainder in each case.

Example 6. On dividing a number by 68, we get 269 as quotient and 0 as remainder on dividing the same number by 67, what will be the remainder ?

Sol. Number = 269 × 68 + 0 = 18292

```
67) 18292 (273        269
    134               ×68
    ---              ----
     489             2152
     469            1614×
     ---            -----
      202           18292
      201
      ---
        1
```

SMART WAY

As per the question the number is a multiple of 68 so on dividing 68m by 67 we always gets remainder as 1.

∴ Required Remainder = 1

Example 7. What least number must be subtracted from 1672 to obtain a number which is completely divisible by 17 ?

Sol.

```
17) 1672 (98
    153
    ---
     142
     136
     ---
       6
```

Number to be subtracted = 6

Example 8. What least number must be subtracted from 13601, so that number is divisible by 87 ?

Sol.

```
87) 13601 (156
    87
    ---
     490
     435
     ---
      551
      522
      ---
       29
```

$\therefore$ Required number = 29

Example 9. What is the smallest 5-digit number exactly divisible by 41 ?

Sol. The smallest 5-digit number = 10000

$\therefore$ Required number = 10000 + (41 – 37) = 10004

```
41) 10000 (243
     62
    ---
     180
     164
     ---
      160
      123
      ---
       37
```

Sum of all the first N natural numbers

S = 1 + 2 + 3 + ----------------- N

$$S = N\frac{(N+1)}{2}$$

Sum of first N odd No.s

S = 1 + 3 + 5 + ----------------- N

$S = N^2$ (N = No. of terms)

Sum of first N even No.s

S = 2 + 4 + 6 + ----------------- N

S = N(N + 1)

Sum of squares of first N natural No.s

$S = 1^2 + 2^2 + 3^2$ ------------------N^2

$$S = \frac{N(N+1)(2N+1)}{6}$$

CONCEPT OF A.P. & G.P.

Arithmetic Series :–

An Arithmetic series is one in which successive numbers are obtained by adding (or subtracting) a fixed number to previous no.

for *e.g.,* 3, 5, 7, 9, 11----------

d (common difference) = 5 – 3 = 2

= 7 – 5 = 2

$a \rightarrow$ first term $n \rightarrow$ number of terms

$l \rightarrow$ last term $= a + (n - 1)d$

sum of n terms $= \frac{n}{2}[a + l]$

Geometric Series :-

Geometric Series is one in which each successive number is obtained by multiplying (or dividing) a fixed number by the previous number.

For example 4, 8, 16, 32, 64-----------

r (common ratio) = $\frac{16}{8} = \frac{8}{4} = 2$

$a \rightarrow$ first term

Sum of n terms = $\frac{a(r^n - 1)}{r - 1}$

Example 10. How many natural numbers between 17 and 80 are divisible by 6 ?

Sol. These numbers are 18, 24, 30, 36,...., 78 which is an A.P.

Here, $a = 18$, $d = 24 - 18 = 6$ and $l = 78$

$l = a + (n - 1) d$

$78 = 18 + (n - 1) 6$

$\Rightarrow (n - 1) \times 6 = 60 \Rightarrow n - 1 = 10 \Rightarrow n = 11$

$\therefore$ Required numbers = 11

> **SMART WAY**
> Series is 18, 24, 30, 36------78
> Your Series can be written as 6 (3, 4, 5, 6----13)
> Total numbers are = 13 – 3 + 1 = 11
> Alsways remember to add 1.

Example 11. Find the sum of all even natural numbers less than 75.

Sol. Sum = 2 + 4 + 6 + ... + 74 which is an A.P.

Here, $a = 2$, $d = 4 - 2 = 2$, $l = 74$, $n = 37$

$\therefore$ Sum = $\frac{n}{2}(a + l) = \frac{37}{2} \times (2 + 74) = 37 \times 38 = 1406$

Example 12. What is the sum of the numbers between 1 to 500 ?

Sol. Sum of numbers from 1 to 500 = $\frac{500\,(500 + 1)}{2}$ [$\because$ Sum of numbers from 1 to $n = \frac{n\,(n+1)}{2}$]

$= \frac{500 \times 501}{2}$

$= \frac{250500}{2} = 125250$

Example 13. Find the sum $(2 + 2^2 + 2^3 + 2^4 + ... + 2^{10})$

Sol. This is an G.P.

$$S = \frac{a\,(r^n - 1)}{r - 1} = \frac{2\,(2^{10} - 1)}{2 - 1} = 2 \times 1023 = 2046$$

NUMBER OF FACTORS OF A COMPOSITE NUMBER

In general, for any composite number C, which can be expressed as $C = a^m \times b^n \times c^p$ ------------ where a, b, c---------are all prime factors and m, n, p ---------------- are positive integers, the number of factors is equal to $(m + 1)(n + 1)(p + 1)$------------------- .

Number of prime factors = $m + n + p$ --------------

Example 14. Find the total number of prime factors in the product $\{(4)^{11} \times 7^5 \times (11)^2\}$.

Sol. $\{(4)^{11} \times 7^5 \times (11)^2\} = (2 \times 2)^{11} \times 7^5 \times (11)^2$

$= (2^2)^{11} \times 7^5 \times (11)^2$

$= 2^{22} \times 7^5 \times 11^2$

$\therefore$ Number of prime factors = (22 + 5 + 2) = 29

CONCEPT OF UNIT DIGIT

Rule (i) For odd No.

When there is an odd digit in the unit place (except 5) multiply the no. by itself until you get 1 in the unit place.

$(---1)^n = (---1)$, $(---3)^{4n} = (---1)$ $(---7)^{4n} = (---1)$

Rule (ii) For even No.

When there is an even digit in the unit place multiply the no. by itself until you get 6 in the unit place.

$(---2)^{4n} = (---6)$, $(---4)^{2n} = (---6)$ $(---6)^n = (---6)$, $(---8)^{4n} = (---6)$

Note : If there is 1, 5 or 6 in the unit place of the given number, then after any times of multiplication, it will have the same digit in the unit place i.e.,

$(---1)^n = (---1)$, $(---5)^n = (---5)$ $(---6)^n = (---6)$

Example 15. Find the remainder when 2^{31} is divided by 5.

Sol. $2^{31} = (2^{10} \times 2^{10} \times 2^{10}) \times 2 = (2^{10})^3 \times 2 = (1024)^3 \times 2$

Unit digit in $\{(1024)^3 \times 2\} = 4 \times 2 = 8$

Now, 8 when divided by 5 gives 3 as remainder.

$\therefore$ 2^{31} when divided by 5 gives remainder = 3

SMART WAY

2^{31} can be written as 2^{28+1+2} or 2^{28+1}. 2^2 the unit digit of 2^{28+1} is 2

2.2^2 the unit digit becomes 8 so on dividing 8 by 5 we get 3 as remainder

Example 16. What is the unit digit in $\{(264)^{102} + (264)^{103}\}$?

Sol. $(264)^{102} + (264)^{103} = (264)^{102}\,[1 + 264]$

$= (264)^{102} + 265$

$\therefore$ Unit digit in $[(4)^{102} \times 5]$

$= [(4^4)^{25} \times 4^2 \times 5]$

$= (6 \times 6 \times 5) = 0$

SMART WAY

$(264)^{102}+(264)^{103}$

we have to find out the unit digit of

$(----4)^{102} + (----4)^{103} = (----6) + (----4)$

your answer is 6 + 4 = 10 the unit digit is 0.

ALGEBRAIC FORMULAE

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(v) $(a + b)^2 - (a - b)^2 = 4ab$

(vi) $(a + b)^2 = (a - b)^2 + 4ab$

(vii) $(a - b)^2 = (a + b)^2 - 4ab$

(viii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(ix) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(x) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(xi) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(xii) If $(a + b + c) = 0$ then $a^3 + b^3 + c^3 = 3abc$

(xiii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2) = (a + b)^3 - 3ab(a + b)$

(xiv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2) = (a - b)^3 + 3ab(a - b)$

Example 17. 5793405 × 9999 = ?

Sol. $5793405 \times 9999 = 5793405 \times (10000 - 1)$

$= 57934050000 - 5793405$

$= 57928256595$