# IBT
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# Quantitative Aptitude

COURSE BOOK

# INDEX

## CLASS TESTS

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**1**

# Number System

**Numbers :** A number is denoted by a group of digits, called numeral.

For denoting a numeral 843215696 can be represented as

| Ten Crores | Crores | Ten Lacs (Millions) | Lacs | Ten Thousands | Thousands | Hundreds | Tens | Units |
|---|---|---|---|---|---|---|---|---|
| $10^8$ | $10^7$ | $10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |
| 8 | 4 | 3 | 2 | 13 | 5 | 6 | 9 | 6 |

## TYPES OF NUMBERS

1. **Natural Numbers :** Counting numbers are called natural numbers.

$$N = \{1, 2, 3, 4, 5, ....\}$$

2. **Whole Numbers :** All counting numbers and 0 form the set of whole numbers.

$$W = \{0, 1, 2, 3, 4, 5, ...\}$$

3. **Integers :** All counting numbers, zero and negative of counting numbers form the set of Integers.

$$I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, ....\}$$

4. **Even Numbers :** The number which is divisible by 2 is called even number. *e.g.,* 2, 4, 12, 28 etc.

5. **Odd Numbers :** The number which is not divisible by 2 is called odd number. *e.g.,* 1, 3, 5, 7 etc.

6. **Prime Numbers :** A number is called a prime number if it has exactly two factors, namely itself and 1. *e.g.,* 2, 5, 11, 19, 23 etc.

7. **Composite number :** The natural number which are not prime, are called composite numbers. *e.g.,* 4, 9, 15, 18, 27 etc.

8. **Rational Numbers :** A rational number is a number that can be put in the form $\dfrac{p}{q}$ where $p$ and $q$ are both integers and $q \neq 0$ *e.g.,* $7, \dfrac{-9}{5}, \dfrac{-2}{7}, \dfrac{1}{4}, 0$ etc.

9. **Irrational Numbers :** An irrational number is a number that cannot be put in the form $\dfrac{p}{q}$ where $p$ and $q$ are both Integers and $q \neq 0$ *e.g.,* $\sqrt{7}, \sqrt{11}, \sqrt{13}$ etc.

10. **Real Numbers :** All those numbers which are either rational or irrational. *e.g.,* $\dfrac{12}{17}, \dfrac{19}{21}, \sqrt{5}, 5+\sqrt{3}$ etc.

11. **Twin Primes :** Two prime numbers which differ by 2 are called twin primes. *e.g.,* 3, 5 ; 5, 7 ; 11, 13 ; are some pairs of twin primes.

## NUMBER TREE
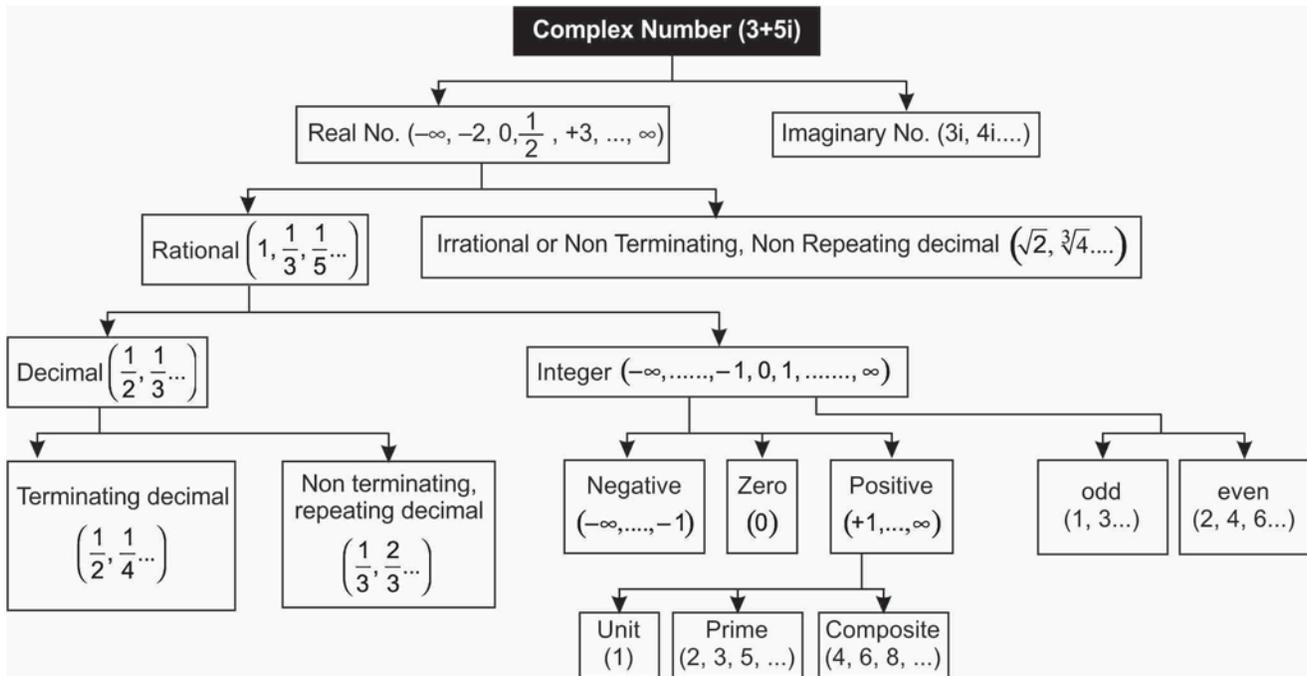


## TESTS FOR DIVISIBILITY OF NUMBERS

(i) **Divisibility by 2 :** A number is divisible by 2 if its units digit is 0, 2, 4, 6 or 8 *e.g.,* 130, 244, 566, 278 etc.

(ii) **Divisibility by 3 :** A number is divisible by 3 if the sum of its digits is multiple of 3.

*e.g.,* (a) 123 : 1 + 2 + 3 = 6 which is the multiple of three hence the number is divisible by 3.

(b) 89612 : 8 + 9 + 6 + 1 + 2 = 26 = 2 + 6 = 8 is not a multiple of three hence the number is not divisible by 3.

(iii) **Divisibility by 4 :** If the number formed by last two digits is divisible by 4. *e.g.,* 1132, 1312, 1400, 1348 etc.

(iv) **Divisibility by 5 :** If number unit's digits is either 0 or 5. *e.g.,* 100, 205, 315 etc.

(v) **Divisibility by 6 :** If number is divisible by both 2 and 3. *e.g.,* 54, 96 etc.

(vi) **Divisiblility by 7 :** For 7 we need to have – osculator – 2.

*e.g.,* 112 divisible by 7 ?

**Step 1.** 11 2 = 11 – 2 × 2 = 7 as 7 is divisible by 7 the number is also divisible by 7.

**Try for :** 2961 divisible by 7 ?

(vi) **Divisibility by 8 :** If number formed by its last three digits is divisible by 8. *e.g.,* 1864, 1024, 2008 and 5000 etc.

(vii) **Divisibility by 9 :** If the sum of numbers digits is a multiple of 9. *e.g.,* 23409, 454554, 66636 etc.

**Example 1.** Find the least value of * for which 7 * 5462 is divisible by 9.

**Sol.** Let the required value be $p$ then

(7 + p + 5 + 4 + 6 + 2) = (24 + p) is divisible by 9.

∴ $p = 3$

**Example 2. If the number 653 $xy$ is divisible by 90, then find the value of $(x + y)$ ?**

**Sol.** $90 = 10 \times 9$

Clearly 653 $xy$ is divisible by 10, so $y = 0$

Now, 653 $x$ 0 is divisible by 9

So, $(6 + 5 + 3 + x + 0) = (14 + x)$ is divisible by 9

So, $x = 4$

∴ $x + y = 4 + 0 = 4$

**(viii) Divisibility by 10 :** If number's unit's digit is zero. *e.g.,* 50, 80, 100, 1310 etc.

**(ix) Divisibility by 11 :** If the difference of the sum of no's digits in even places and the sum of its digits in odd places is either 0 or a multiple of 11. *e.g.,* 909183, 540045, 184712 etc.

## FACTS ABOUT ODD AND EVEN NUMBERS

| | |
|---|---|
| odd $\pm$ odd = even | odd $\times$ odd = odd |
| odd $\pm$ even = odd | odd $\times$ even = even |
| even $\pm$ even = even | even $\times$ even = even |

## FORMULA FOR DIVISION OF WHOLE NUMBERS

Dividend = Divisor $\times$ Quotient + Remainder

**Example 3. A number when divided by 899 gives a remainder of 63. If the same number is divided by 29, then what will be the remainder ?**

**Sol.** 
$$\text{Number} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$
$$= 899 \times x + 63$$
$$= 31 \times 29 \times x + 29 \times 2 + 5$$
$$= 29(31x + 2) + 5$$

$\therefore$ The remainder when the number is divided by 29 is 5

> **SMART WAY**
>
> Number = Divisor × Quotient + Remainder
> = 899 × 1 + 63 = 962
> (Always take Quotient as 1)
> No. Divide by 29 we get remainder as 5)

**Example 4. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, determine the dividend.**

**Sol.** Let the quotient be Q and the remainder be R

$\therefore$ 
$$\text{Divisor} = 5 \times 46 = 230$$
$$\text{Quotient} = \frac{230}{10} = 23$$

$\therefore$ 
$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$
$$= 230 \times 23 + 46 = 5290 + 46 = 5336$$

> **SMART WAY**
>
> As per the question the number is a multiple of 68 so on dividing 68m by 67 we always gets remainder as 1.

**Example 5. Find the largest number which divides 25, 73 and 97 leaving an equal remainder in each case.**

**Sol.** $\because$ 
$$\text{Number} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$
$$25 = 24 \times 1 + 1 \qquad \qquad ...(i)$$
$$73 = 24 \times 3 + 1 \qquad \qquad ...(ii)$$
$$97 = 24 \times 4 + 1 \qquad \qquad ...(iii)$$

$\therefore$ 24 is the largest number which divides the given three numbers leaving 1 as remainder in each case.

**Example 6. On dividing a number by 68, we get 269 as quotient and 0 as remainder on dividing the same number by 67, what will be the remainder ?**

**Sol.** Number = $269 \times 68 + 0 = 18292$

```
        67) 18292 (273
            134
            ‾‾‾‾
            489
            469
            ‾‾‾‾
            202
            201
            ‾‾‾‾
              1
```

$$
\begin{array}{r}
269 \\
\times 68 \\
\hline
2152 \\
1614\times \\
\hline
18292
\end{array}
$$

$\therefore$ Required Remainder = 1

**Example 7. What least number must be subtracted from 1672 to obtain a number which is completely divisible by 17 ?**

**Sol.**

$$17) \overline{1672} (98$$
$$\underline{153}$$
$$142$$
$$\underline{136} \qquad \text{Number to be subtracted} = 6$$
$$\underline{6}$$

**Example 8. What least number must be subtracted from 13601, so that number is divisible by 87 ?**

**Sol.**
$$87) \overline{13601} (156$$
$$\underline{87}$$
$$490$$
$$\underline{435}$$
$$551$$
$$522$$
$$\underline{29} \qquad \therefore \text{ Required number} = 29$$

**Example 9. What is the smallest 5-digit number exactly divisible by 41 ?**

**Sol.** The smallest 5-digit number = 10000

$\therefore$ Required number = 10000 + (41 – 37) = 10004

$$41) \overline{10000} (243$$
$$\underline{62}$$
$$180$$
$$\underline{164}$$
$$160$$
$$\underline{123}$$
$$37$$

## CONCEPT OF UNIT DIGIT

**Rule (i) For odd No.**

When there is an odd digit in the unit place (except 5) multiply the no. by itself until you get 1 in the unit place.

$(---1)^n = (---1)$, $(---3)^{4n} = (---1)$ $(---7)^{4n} = (---1)$

**Rule (ii) For even No.**

When there is an even digit in the unit place multiply the no. by itself until you get 6 in the unit place.

$(---2)^{4n} = (---6)$, $(---4)^{2n} = (---6)$ $(---6)^n = (---6)$, $(---8)^{4n} = (---6)$

**Note :** If there is 1, 5 or 6 in the unit place of the given number, then after any times of multiplication, it will have the same digit in the unit place i.e.,

$(---1)^n = (---1)$, $(---5)^n = (---5)$ $(---6)^n = (---6)$

**Example 10. Find the remainder when $2^{31}$ is divided by 5.**

**Sol.** $2^{31} = (2^{10} \times 2^{10} \times 2^{10}) \times 2 = (2^{10})^3 \times 2 = (1024)^3 \times 2$

Unit digit in $\{(1024)^3 \times 2\} = 4 \times 2 = 8$

Now, 8 when divided by 5 gives 3 as remainder.

$\therefore 2^{31}$ when divided by 5 gives remainder = 3

**Example 11. What is the unit digit in $\{(264)^{102} + (264)^{103}\}$ ?**

**Sol.** $(264)^{102} + (264)^{103} = (264)^{102} [1 + 264]$

$$= (264)^{102} + 265$$

$\therefore$ Unit digit in $[(4)^{102} \times 5] = [(4^4)^{25} \times 4^2 \times 5]$

$$= (6 \times 6 \times 5) = 0$$

**Example 12. 5793405 × 9999 = ?**

**Sol.** $5793405 \times 9999 = 5793405 \times (10000 - 1)$

$$= 57934050000 - 5793405$$

$$= 57928256595$$

| SMART WAY |
|---|
| $2^{31}$ can be written as $2^{28+1+2}$ or $2^{28+1} . 2^2$ the unit digit of $2^{28+1}$ is 2 $2 . 2^2$ the unit digit becomes 8 so on dividing 8 by 5 we get 3 as remainder |

| SMART WAY |
|---|
| $(264)^{102} + (264)^{103}$ we have to find out the unit digit of $(----4)^{102} + (----4)^{103} = (----6) + (----4)$ your answer is 6 + 4 = 10 the unit digit is 0. |

## SOME IMPORTANT RESULTS

(i) The smallest natural number or +ve integer is +1.

(ii) The greatest negative Integer is –1.

(iii) The number '0' is neither +ve nor negative integer.

(iv) 1 is only such number which is neither a prime number nor a composite no.

(v) 2 is only such number which an even number as well as prime no.

(vi) 2 is smallest prime number.

(vii) The number of prime numbers between 1 to 100 is 25.

(viii) The number of prime numbers between 1 to 1000 is 168.

(ix) A square no. may have 0, 1, 4, 5, 6 or 9 in its unit's place.

(x) A cubic no. may have any digit from 0 to 9 in its unit's place.

## LEVEL OF DIFFICULTY–1

1. The difference between the place value and face value of 6 in the numeral 856973 is :
   (a) 973      (b) 6973      (c) 5994      (d) 897
   (e) None of these.

2. The difference between the place values of two sevens in the numeral 69758472 is
   (a) 0      (b) 6993      (c) 699930      (d) 01
   (e) None of these.

3. The unit digit in the product (784 × 618 × 917 × 463) is :
   (a) 2      (b) 3      (c) 4      (d) 10
   (e) 5.

4. If the number 517 * 324 is completely divisible by 3, then the smallest whole number in place of * will be :
   (a) 0      (b) 1      (c) 2      (d) 4
   (e) 6.

5. Which one of the following numbers is completely divisible by 99 ?
   (a) 3572404      (b) 135792      (c) 913464      (d) 114345
   (e) None of these.

6. If the product 4864 × 4p2 is divisible by 12 the least value of $p$ is :
   (a) 2      (b) 5      (c) 6      (d) 7
   (e) None of these.

7. Which one of the following numbers is exactly divisible by 11 ?
   (a) 235641      (b) 245642      (c) 315624      (d) 415624
   (e) None of these.

8. The sum of first five prime numbers is :
   (a) 11      (b) 18      (c) 26      (d) 28
   (e) 326419.

9. What least number must be added to 1056, so that the sum is completely divisible by 23 ?

(a) 2                     (b) 3                     (c) 18                     (d) 21

(e) None of these.

10. Which of the following numbers is divisible by each one of 3, 7, 9 and 11 ?

(a) 639                  (b) 2079                  (c) 3791                  (d) 37911

(e) None of these.

11. On dividing a number by 56, we get 29 as remainder. On dividing the same number by 8, what will be the remainder ?

(a) 4                     (b) 5                     (c) 6                     (d) 7

(e) None of these.

12. A number was divided successively in order by 4, 5 and 6. The remainders were respectively 2, 3 and 4. The number is :

(a) 214                  (b) 476                  (c) 954                  (d) 1908

(e) None of these.

13. The difference of the squares of two consecutive even integers is exactly divisible by which of the following integers ?

(a) 3                     (b) 4                     (c) 6                     (d) 7

(e) None of these.

14. The smallest 6-digit number exactly divisible by 111 is :

(a) 111111              (b) 110011              (c) 100011              (d) 110101

(e) 100111.

15. If the number $(10^n - 1)$ is divisible by 11 then $n$ is :

(a) odd number         (b) even number     (c) any number         (d) multiple of 11

(e) None of these.

16. If $(6)^{15} \times (10)^5 \times (15)^6 = 2^x \times 3^y \times 5^z$

Find the value of $x + y + z$

(a) 26                   (b) 52                   (c) 42                   (d) 48

(e) 45

17. Find the least number by which 19404 must be multiplied so as to make it a perfect square.

(a) 2                     (b) 3                     (c) 7                     (d) 11

(e) None of these

18. A number consists of two digits. The sum of the digit is 6. If 18 are subtracted from the number the digits are interchanged. Find the number.

(a) 48                    (b) 42                    (c) 56                    (d) 81

(e) None of these

## ANSWERS

| 1. (c) | 2. (c) | 3. (a) | 4. (b) | 5. (d) | 6. (e) | 7. (d) | 8. (d) | 9. (a) | 10. (b) |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| 11. (b) | 12. (a) | 13. (b) | 14. (c) | 15. (b) | 16. (b) | 17. (d) | 18. (b) | | |